Subsidizing Uncertain Investments: The Role of Production Technology and Imprecise Learning

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Abstract

This paper explores the interplay between government subsidies, production technology, and learning through signals in shaping a firm's investment strategy. Our real options framework accounts for uncertainty across investment stages and that the informativeness of signals is limited. The optimal subsidization policy aligns a firm's investment incentives with evolving knowledge during the investment process. We demonstrate that the interplay between the nature of the production technology and the quality of information plays a central role. Subsidies are most effective when signals are more informative, especially when the technology payoff depend more on later-stage investments. Our analysis highlights the importance of managing uncertainty at every stage to maximize social net benefits, offering insights for policymakers on structuring subsidies for diverse investment projects under uncertainty.

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1 Introduction

This paper investigates the complex interplay between government subsidies, production technology, and learning through imprecise signals in shaping a firm's investment strategy. We place particular emphasis on understanding how these factors collectively impact social net benefits and project valuation. Our findings reveal that the effectiveness of subsidies is highly sensitive to the interplay between the informativeness of signals received during the investment process and the nature of the production technology. Specifically, more informative signals enhance both governmental and corporate benefits, with this effect becoming more pronounced when the returns of the production technology depend more on later-stage investments. A central aspect of this investigation is the derivation of an optimal subsidization policy that aligns the firm's intrinsic investment incentives with the evolving knowledge gained through investments. Thus, our analysis addresses research questions such as: How do the firm's production technology and the informativeness of investment signals affect the government's subsidy scheme? What role does the interplay between production technology and signal informativeness play in shaping subsidization policy and net benefits?

By integrating these elements, we aim to elucidate how subsidies can be structured to enhance both the private value of the project and its broader societal benefits. Government subsidies of investments play a crucial role in projects that require multiple investment outlays, particularly when the project's viability is uncertain. These subsidies provide firms with a real options value, enabling them to make an initial investment and then reassess their commitment as more information becomes available. This flexibility allows firms to manage risk more effectively while pursuing projects with significant social and private benefits. Our exploration of how government subsidies influence investment decisions highlights the opportunities for firms to adapt their strategies as they learn about the project's potential outcomes. These insights can guide policymakers in determining which investments to subsidize, ensuring that public funds are allocated to projects that maximize both social and private returns.

Our paper relates to the literature on real options and government intervention. Dixit and Pindyck (1994) and Trigeorgis (1996) provide a general overview of real options and their applications. More directly related to our analysis, Pindyck (2000) employs real options to discuss how uncertainty and irreversibility can affect the timing of environmental policies. Azevedo et al. (2021) consider a real options model that analyzes the effect of government's subsidies and taxation policy on the timing and size of investments. Related to our analysis they find that a higher subsidy encourages investment timing in the sense that a fixed subsidy induces smaller size investments, whereas a variable subsidy encourages larger size investments. Hu et al. (2019) analyze how state subsidies to firms affect corporate investment efficiency in a study of Chinese firms. They find that government subsidies have a negative effect on firms' investment efficiency, and this negative effect is more pronounced for firms that are less financially constrained. Subsidies can thus alleviate under-investment problems, but they amplify over-investment problems. Barbosa et al. (2022) examine the effects of various finite-lived subsidies on investment timing and social welfare using a model with a demand function with an exogenous multiplicative shock following a geometric Brownian motion. They find that a finite-lived subsidy policy eliminates the under-investment inefficiency stemming from the monopolistic firm's preferences to invest later than what a social planner would do.

Our paper differs from the previous studies as our focus is not on investment timing but rather on the interplay between the production technology for a multi-stage investment project and limited learning during the investment process. Furthermore, in deriving the subsidization policy we specifically implement a governmental criteria that limits the level of subsidization.

We find that even with a constant Marginal Value of Public Funds ("MVPF"), the social net benefits are highly sensitive to the interplay between learning and the production technology. Specifically, a key outcome of our analysis is that when a firm can learn about a project's viability during a multi-staged investment process, subsidization is particularly valuable when the production technology exhibits a higher elasticity on later-stage investments. This effect comes about despite the fact that we set up a model in which the subsidization policy does not depend on the firm's ability to learn nor the relative elasticity of the investments as such. The intuition for our main result is as follows. First, the government is willing to subsidize a firm's project as long as this generates a marginal value larger than a given threshold. Since the firm is interested in exploiting that and we have decreasing returns to scale, the firm will invest up to the level at which this condition is binding. Second, the firm's ability to learn during the investment process is only valuable if it can exploit this. This requires that the firm has flexibility in the investment levels at the different stages. Third, that flexibility is more valuable for the firm when the elasticity of the later-stage investment is higher than the elasticity of the initial-stage investment. Thus, the government gets the most out of supporting a firm that is intrinsically motivated to exploit its learning to maximize its value.

We investigate sequential investment behavior of privately-owned firms in the presence of investment subsidies and uncertainty. The motivation for this inquiry is two-fold: Firstly, the emergence and growth subsidies for private investments. For instance, The Inflation Reduction Act of 2022 (IRA, 2022), will allocate approximately $369 billion to investments in energy security and climate change.[1] In response to the IRA, European Commission President Ursula von der Leyen announced the EU's "Green Deal Industrial Plan" which seeks to stimulate private investments and retain Europe based green tech firms. With accelerating subsidy initiatives, it is paramount that we develop a better understanding of the dynamics between public and private sector climate commitments.[2] Secondly, many investment projects require multiple capital outlays, which are made under uncertainty. Therefore, to analyze the investment behavior with subsidies, we expand the traditional growth option framework to account for the fact that even late-stage capital outlays may be committed under information uncertainty. We carry out this analysis in a model where investment outlays are complementary.

The observation that information uncertainty is present in all investment stages is of great importance from a modeling point of view. The real options framework is a widely used approach to analyzing investment problems in which capital is provided in stages and with information being revealed over time (e.g., Brennan and Schwartz, 1985;

[1] Estimations by Joint Committee on Taxation and the Congressional Budget Office. Available online via www.democrats.senate.gov (accessed 3/28/2023).

[2] Climate related investments and subsidies have been a motivating factor in the ideation phase of this paper, but our results and analyses apply for a much broader set of subsidized investments.

McDonald and Siegel, 1986). Importantly, for growth options it is commonly assumed that we can identify an observable process that eventually will determine the terminal value of the investment project. When investment outcomes are influenced by external risks (e.g., climate risks) it is important to challenge this notion of observability. Formally, we address this issue as a question of signal accuracy. "Perfect accuracy" can be interpreted as cases with full transparency of project viability before the later-stage investment is made. These cases are equivalent to the exercise decision for a basic call-option with the later-stage investment as strike price. "No accuracy" can be interpreted as cases where the arriving information is of no value and thus the decision problem is equivalent to a basic staged financing problem with complementary investment inputs. The truly interesting and relevant inquiries pertain to cases in between these two extremes. There are various examples where uncertainty is an essential factor. For instance, the decision to set up electric vehicle charging stations is done sequentially and at a pace that reflects local demand. Similarly, the exploration of ocean energy technologies has decades-long time horizons and involves learning about radically new engineering challenges; and stimulating the hydrogen economy for "hard to de-carbonize" sectors such as transportation requires substantial sequential investments by multiple stakeholders in concert. In these examples, initial capital outlays can be regarded as investments in cost-effectiveness, while subsequent expenditures facilitate market development and scaling. The overall value of the project is inherently influenced by the synergy of these efforts. Consequently, we consider inter-temporal investment outlays to be complementary in nature.

We assume that the subsidies are announced prior to any investments are made, and we furthermore assume that this announcement is considered credible.[3] For simplicity, we consider subsidies as fractions of the actual investment outlays. These are determined by the government's incentive to maximize the expected social value of the policy, constrained by a lower bound on the MVPF.[4] The use of MVPF for policy decisions like these is well established (e.g., Mayshar, 1990; Slemrod and Yitzhaki, 1996, 2001; Kleven and Kreiner,

[3]We discuss potential extensions where this assumption is relaxed in Section 4.

[4]This setup resembles a decision maker who maximizes a social net present value, subject to an internal hurdle rate.

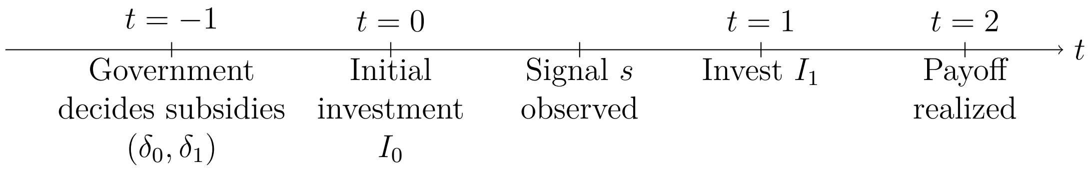

Figure 1: **Timeline of the model.** The government initially sets a subsidy policy (δ_0, δ_1). The firm responds with its investment policy (I_0, I_1) where the later-stage investment depends on the signal s.

2006).

The paper is organized as follows: Section 2 presents the model and its solution. Section 3 investigates various economic implications and derives the option value associated with the investment problem. Section 4 explores several implications and extensions of our setting and Section 5 concludes. All proves are presented in an appendix.

2 The model

A firm can adopt a project that requires a two-stage investment from which an uncertain return can be gained, and the firm targets to set investment levels to maximize its expected value at each stage. The investments are complementary. That is, if the firm does not invest in the first stage, it cannot make the project profitable and thus no later-stage investment will occur. Similarly, even with a positive initial investment, without a later-stage investment, no return will materialize. If the firm makes the initial investment, it obtains a signal regarding the value of the final project. However, the signal is not fully informative about the productivity of invested capital and this dampens the firm's incentive to invest.

The government can offer subsidies to encourage the firm to undertake further investments. We assume that these subsidies are set to maximize the expected social net benefits, subject to a minimum required MVPF.[5] The firm and the government are assumed to be risk neutral and both have zero discounting rates. The aim of our analysis is to investigate

[5] See for instance Finkelstein and Hendren (2020) and Hendren and Sprung-Keyser (2020) for current and comprehensive treatments.

how the interplay between subsidies and imprecise signals affects the firm's investment policy and hence to understand how this interplay effects social net benefits and the value of the project. A key step in this analysis is to derive the optimal subsidization policy taking the firm's intrinsic investment incentive and learning into account. Figure 1 illustrates the timeline. We turn to the details of the model below.

2.1 Investments, uncertainty, and value

The investment opportunity requires two capital outlays, I_0 and I_1. The investments yield a terminal value of

$$V = xI_0^{\beta_0} I_1^{\beta_1}, \tag{1}$$

where $x \in \{0, k\}$ is the uncertain total productivity of capital and $k > 0$ is a level parameter. β_i is the elasticity of capital outlay I_i for $i \in \{0, 1\}$. The elasticities play a central role for the firm's incentive to invest in the different stages. Hence, they are important for the government to take into account when deciding on a subsidy policy. We assume decreasing returns to scale of capital investments and that $\beta_0 + \beta_1 < 1$.

The productivity of capital is either high or low. The investment is only valuable in the former case, and we let the ex ante belief of this be

$$q = \mathbb{P}\left(x = k\right). \tag{2}$$

To focus on the role of subsidies as a means to having the firm increasing its investments, we do not impose any budgetary constraints in the firm's investment decision problem. The unconstrained firm has an optimal investment policy with finite investments in the two stages since capital has a decreasing returns to scale. Note that if capital is known to be productive, that is, $q = 1$, then standard first-order conditions give us

$$I_0|_{q=1} = \left[k\beta_0^{1-\beta_1}\beta_1^{\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{3}$$

and

$$I_1|_{q=1} = \frac{\beta_1}{\beta_0} I_0|_{q=1}. \tag{4}$$

This clearly illustrates the direct role of the level parameter and the elasticities of capital. It also highlights the relation between initial and later-stage investments. However, the above investments assume full knowledge about the growth potential of the investment. This is not so in practice and that provides a challenge for society.

When the total productivity of capital is uncertain, firms are much less willing to undertake the desired investments. We consider two elements that impact the firm's investment policy. The first element is that there is a signal about the total productivity of capital prior to the later-stage investment. The second element is through governmental subsidization of investments.[6]

2.2 Information, signals, and accuracy

We first consider the signal. Prior to the final capital outlay, I_1, the firm observes a signal, s, about the likelihood that the investment will be valuable. To simplify the analysis, we assume that the signal is binary, i.e., $s \in \{L, H\}$ (low or high). In Appendix 6.8 we show that our results carry over to a setting in which the signal has a continuous state space. In our present setup with a binary signal we think of the signal as, for example, evidence that the technology is feasible. On the other hand, it could also be evidence that the technology will be ineffective. In either case, the signal provides valuable information to the firm prior to the later-stage investment. We assume that the signal is unbiased in the sense that a high signal when the productivity of capital is high, is as likely as a low signal when the productivity of capital is low (Casamatta and Haritchabalet, 2007; Flor and Grell, 2013). That is,

$$\mathbb{P}\left(s = H | x = k\right) = \mathbb{P}\left(s = L | x = 0\right) \triangleq \alpha \in \left[1/2; 1\right]. \tag{5}$$

[6]Of course, one can think of cases in which the government dictates which technology is the one to invest in as well as how much a firm has to invest. However, we consider a market-based approach and let the government incentivize firms to consider various alternative investment levels.

In the following we need to consider probabilities for several different events. To ease notation, we will henceforth write $p(H|k)$ instead of $\mathbb{P}(s = H|x = k)$, and similarly $p(L|k)$, $p(H|0)$, and $p(L|0)$. We will also write $p(k|H)$ instead of $\mathbb{P}(x = k|s = H)$, and similarly for $p(0|H)$, $p(k|L)$, and $p(0|L)$. Finally, we will use p_H and p_L instead of $\mathbb{P}(s = H)$ and $\mathbb{P}(s = L)$. Formally, we have:

$$p_H = 1 - (\alpha + q) + 2\alpha q, \tag{6}$$

$$p(k|H) = \frac{\alpha q}{1 - (\alpha + q) + 2\alpha q}, \tag{7}$$

$$p(k|L) = \frac{(1 - \alpha)q}{(\alpha + q) - 2\alpha q}. \tag{8}$$

We assume that α is public knowledge before any subsidies are announced and before any investments are made. Importantly, we can think of α in (5) as a parameter that controls the informativeness (or accuracy) of the signal. We specify this in the following lemma.

Lemma 1 *The signal $s \in \{L, H\}$ is informative about the total productivity $x \in \{0, k\}$, in the sense that $p(k|H) > p(k|L)$ and $p(0|L) > p(0|H)$ if and only if $\alpha > 1/2$. The signal is uninformative in the sense that $q = p(k|H) = p(k|L)$ and $1 - q = p(0|H) = p(0|L)$ if and only if $\alpha = 1/2$.*

Lemma 2 *Given the policy (δ_0, δ_1), the optimal initial investment, I_0^*, is given as:*

$$I_0^* = \left[k\left(\frac{\beta_0}{1-\delta_0}\right)^{1-\beta_1}\left(\frac{\beta_1}{1-\delta_1}\right)^{\beta_1} g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{9}$$

where

$$g = g(\alpha, q, \beta_1) \triangleq \mathbb{E}[p(k|s)^{\frac{1}{1-\beta_1}}] = p_H p(k|H)^{\frac{1}{1-\beta_1}} + p_L p(k|L)^{\frac{1}{1-\beta_1}}, \tag{10}$$

and the optimal later-stage investment, $I_1^*(s)$, is given as

$$I_1^*(s) = \frac{p(k|s)^{\frac{1}{1-\beta_1}}}{g} \cdot \frac{\beta_1}{\beta_0} \cdot \frac{1-\delta_0}{1-\delta_1} \cdot I_0^*. \tag{11}$$

Consequently, these investments yield a terminal value of:

$$V(x, s) = \frac{x}{k} \cdot \frac{p(k|s)^{\frac{\beta_1}{1-\beta_1}}}{g} \cdot \frac{1-\delta_0}{\beta_0} \cdot I_0^*. \tag{12}$$

Corollary 1 *The expected later-stage investment is given as:*

$$\mathbb{E}[I_1^*] = \frac{\beta_1}{\beta_0} \cdot \frac{1-\delta_0}{1-\delta_1} \cdot I_0^*. \tag{13}$$

The investment levels from Lemma 2 yields an expected terminal value of

$$\mathbb{E}[V] = \frac{1-\delta_0}{\beta_0} \cdot I_0^* = \frac{1-\delta_1}{\beta_1} \cdot \mathbb{E}[I_1^*]. \tag{14}$$

The government wants to maximize social net benefits subject to the constraint that the MVPF is sufficiently high. We measure social net benefits as the difference between social benefits and social costs. The social benefits are given as the sum of the market value of the firm and the spillover effects from the firm's outcome and investments. The social costs are given as the total expected investment amount. The spillover effect of outcome is measured as a fraction of the firm's market value, γ_V. The spillover effect of investment is measured as a fraction of the firm's investments, $\gamma_I \in [0, 1[$. Hence, the social net benefits are measured as

$$SNB = (1+\gamma_V)\mathbb{E}[V] + \gamma_I(I_0 + \mathbb{E}[I_1]) - (I_0 + \mathbb{E}[I_1]). \tag{15}$$

To calculate the MVPF, we consider the ratio of externality benefits of subsidies relative to the costs of subsidies. Using the above spillover effects, the externality benefits of subsidies are measured as

$$SubBen = \gamma_V \mathbb{E}[V] + \gamma_I(I_0 + \mathbb{E}[I_1]), \tag{16}$$

and the expected costs of subsidies are

$$SubCost = \delta_0 I_0 + \delta_1 E[I_1]. \tag{17}$$

Consequently, we measure the government's MVPF as the ratio

$$MVPF = \frac{SubBen}{SubCost}, \tag{18}$$

and the government's constraint is that MVPF is higher than a level denoted $m > 0$.

Therefore, the government's problem is:

$$\max_{\delta_0,\delta_1} \quad (1+\gamma_V)\mathbb{E}[V] - (1-\gamma_I)(I_0 + \mathbb{E}[I_1]) \tag{19}$$

s.t.

$$\tag{20}$$

$$\frac{\gamma_V \mathbb{E}[V] + \gamma_I (I_0 + \mathbb{E}[I_1])}{\delta_0 I_0 + \delta_1 \mathbb{E}[I_1]} \geq m. \tag{21}$$

Lemma 3 *If the government seeks to maximize the expected social net benefits such that the MVPF stays above a predetermined threshold, m, the optimal investment subsidy (δ_0^*, δ_1^*) is given by:*

$$\delta_0^* = \delta_1^* = \frac{\gamma_V + \gamma_I(\beta_0 + \beta_1)}{\gamma_V + m(\beta_0 + \beta_1)} \triangleq \delta^*, \tag{22}$$

where $0 \leq \gamma_I < m$ implies that $\delta \in [0; 1]$.

Note that our model implies that a very lax threshold for the MVPF leads to full subsidization of investments. That is, as m goes to 0 – thus also forcing γ_I to 0 – then the subsidy rate converges to unity. In contrast, a stricter threshold leads to a lower subsidy rate and eventually no subsidization at all. Furthermore, as can be seen in Proposition 1 below, the subsidy rate increases in each of the spill-over effects (γ_V and γ_I), which is to be expected, since the subsidy increases the overall investment levels as well as the expected terminal value of the project. Interestingly, the subsidy rate does not depend on the individual relative elasticities of capital, but it depends on the sum $(\beta_0 + \beta_1)$. This is very likely due to the fact that no renegotiation of the subsidies is possible and that the government's subsidy policy is independent of the arrival of new information. As discussed in Section 4 this and other extensions of the base model lead to many interesting observations about the investment ratios, i.e. the ratio between the expected later-stage and the initial investment as well as the ratio between actual later-stage investment for $s = H$ vs. $s = L$.[7]

[7] As indicated in Section 4, allowing the subsidy rate, $\delta(s)$, to be flexible will have non-trivial impacts on the firm's investment policy. For instance, if the spill-over effects from the investments themselves, γ_I, are relatively high, it is possible that the subsidy rate will be set such that the firm is over-compensated for it capital commitment. The firm, in turn, can take this mechanism into account and invest more

Proposition 1 *The optimal subsidy policy, δ^*, is increasing in γ_V and γ_I, decreasing in β_0, β_1, and m, and is independent of all other model parameters. Specifically,*

$$\frac{\partial \delta^*}{\partial \gamma_V} = \frac{(m-\gamma_I)(\beta_0+\beta_1)}{(\gamma_V + m(\beta_0+\beta_1))^2} > 0, \tag{23}$$

$$\frac{\partial \delta^*}{\partial \gamma_I} = \frac{\beta_0+\beta_1}{m(\beta_0+\beta_1)+\gamma_V} \in [0;1], \tag{24}$$

$$\frac{\partial \delta^*}{\partial \beta_i} = -\frac{\gamma_V(m-\gamma_I)}{(m(\beta_0+\beta_1)+\gamma_V)^2} < 0, \quad i \in \{0,1\} \tag{25}$$

$$\frac{\partial \delta^*}{\partial m} = -\frac{(\beta_0+\beta_1)(\gamma_V + (\beta_0+\beta_1)\gamma_I)}{(m(\beta_0+\beta_1)+\gamma_V)^2} < 0. \tag{26}$$

With the optimal subsidy policy in place we can derive the equilibrium investment levels and terminal value of the project:

Theorem 1 *The equilibrium investment levels are given by:*

$$I_0^* = \left[\frac{\gamma_V + m(\beta_0+\beta_1)}{(m-\gamma_I)(\beta_0+\beta_1)} k \beta_0^{1-\beta_1} \beta_1^{\beta_1} g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{27}$$

and

$$I_1^*(s) = \frac{p(k|s)^{\frac{1}{1-\beta_1}}}{g} \cdot \frac{\beta_1}{\beta_0} \cdot I_0^*. \tag{28}$$

Consequently, these investments yield a terminal value of:

$$V(x,s) = \frac{x}{k} \cdot \frac{p(k|s)^{\frac{\beta_1}{1-\beta_1}}}{g} \cdot \frac{1-\delta^*}{\beta_0} \cdot I_0^*. \tag{29}$$

From Theorem 1 we can obtain the expected value of the later-stage investment and we denote the total expected investment level as $\Psi(\alpha, \delta^*)$. This allows us to write the value of the firm on a convenient form:

Corollary 2 *The expected later-stage investment is $\mathbb{E}[I_1^*] = \frac{\beta_1}{\beta_0} I_0^*$ and the total expected investment level is*

$$\Psi(\alpha, \delta^*) = \frac{\beta_0+\beta_1}{\beta_0} \left[\frac{1}{1-\delta^*} k \beta_0^{1-\beta_1} \beta^{\beta_1} g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}}. \tag{30}$$

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than it would in the base model. The opposite effect may occur if the market value spill-over effect, γ_V, dominates.

Table 1: Base case parameters for the numerical analysis

Production scaling, k	100	Information quality, α	0.6
Elasticity of initial investments, β_0	0.2	MVPF constraint, m	1.2
Elasticity of later-stage investments, β_1	0.4	Spillover effect of outcome, γ_V	0.4
Probability technology is viable, q	0.15	Spillover effect of investments, γ_I	0.2

It follows that

$$I_0^* = \frac{\beta_0}{\beta_0 + \beta_1}\Psi(\alpha, \delta^*), \textit{ and } \mathbb{E}[I_1^*] = \frac{\beta_1}{\beta_0 + \beta_1}\Psi(\alpha, \delta^*), \tag{31}$$

and the expected terminal value is

$$\mathbb{E}[V] = \left[k\left(\frac{\gamma_V + m(\beta_0 + \beta_1)}{(m - \gamma_I)(\beta_0 + \beta_1)}\right)^{\beta_0 + \beta_1} \beta_0^{\beta_0}\beta_1^{\beta_1} g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}} = \frac{1 - \delta^*}{\beta_0 + \beta_1}\Psi(\alpha, \delta^*). \tag{32}$$

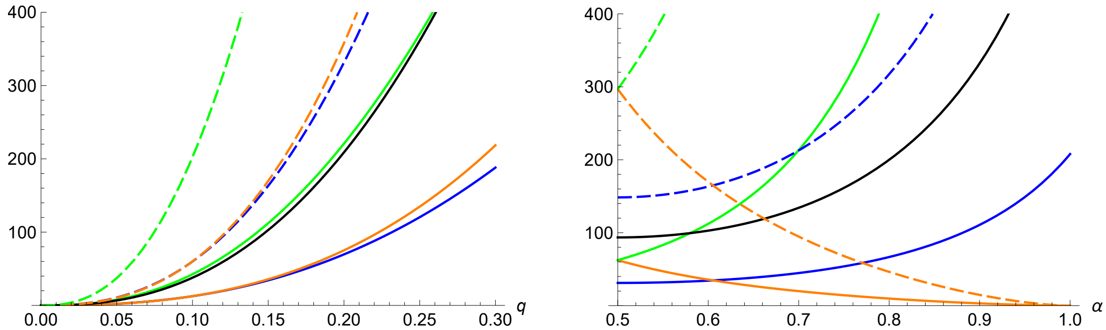
2.3 Analysis of investment effects

Information affects the firm's investment policy through two channels, the ex ante probability that the technology is viable, q, and the information quality, α. We analyze the effects of these in Figure 2 based on the base case in Table 1. Intuitively, the higher the prior success probability is, the higher is the firm's interest in investing in the technology. Figure 2(a) illustrates that investments also without subsidies increase in q. In particular we observe that the later-stage investment after a low signal increases a lot when q increases.

The other channel affecting the investment policy is the information quality, α. This affects the importance of the signal the firm receives prior to deciding on its later-stage investments. Since a higher α makes the signal more precise, there is a natural relationship between α, the probability of receiving a high signal, and the prior probability. For example, if $q = 0.5$, then the probability of a high signal is also 0.5 independently of α. Formally, we have

$$\frac{\partial p_H}{\partial \alpha} = 2q - 1 \qquad \text{and} \qquad \frac{\partial p_L}{\partial \alpha} = 1 - 2q = -\frac{\partial p_H}{\partial \alpha}. \tag{33}$$

Importantly, a higher α increases the conditional probability of a viable technology, if the signal is high, and it decreases the conditional probability, if the signal is low. Specifically,



(a) Effect of q.　　(b) Effect of α.

Figure 2: **Effects of information parameters**. Expected investments with and without subsidies. The blue curve is the initial investment, I_0. The orange (green) curve is the later-stage investment following a low (high) signal. The black curve is total expected investments, $\Psi(\alpha, 0)$. The dashed curves depict investments using the optimal subsidy rate δ^*. The panels use the base case from Table 1.

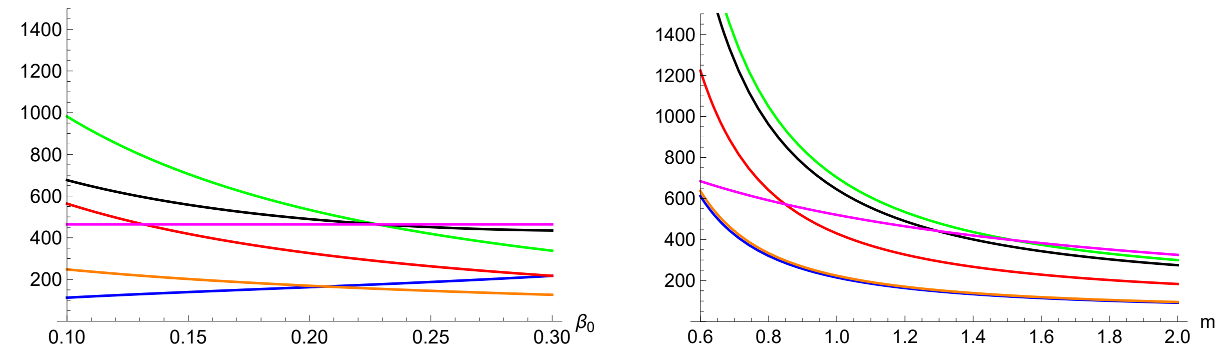
we can show that

$$\frac{\partial p(k|H)}{\partial \alpha} = \frac{q(1-q)}{p_H^2} > 0 \qquad \text{and} \qquad \frac{\partial p(k|L)}{\partial \alpha} = \frac{-q(1-q)}{p_L^2} < 0. \tag{34}$$

This illustrates why we often interpret a higher α as a more informative signal. It seems intuitive that a more accurate signal is beneficial for the firm. Indeed, we can establish that $g(\alpha, q, \beta_1)$ from Lemma 2 is increasing in the information accuracy.

Lemma 4 *$g = g(\alpha, q, \beta_1)$ defined in Lemma 2 is strictly increasing in α and q, while decreasing in β_1.*

It follows from Theorem 1 that a more precise signal increases the initial investment and that the later-stage investments are more sensitive in the signal. These effects are illustrated in Figure 2(b). The intuition is similar to that of the prior success probability, but with more pronounced effects, in particular for the later-stage investments. Intuitively, a higher α increases the value of the firm's flexibility to adapt investments to unraveling information whereas a higher q does not help the firm to exploit investment flexibility. Although the success probability does not affect the subsidization policy, δ^*, the introduction of subsidies impacts the firm's investments substantially. That is, if the firm can obtain subsidies the impact of the above-mentioned effects will be larger in absolute terms. However, as the optimal subsidization policy δ^* is independent of q and α, the effects are due



(a) Investment for varying β_0. $\beta_1 = 0.6 - \beta_0$.

(b) Investment for varying m.

Figure 3: **Investments with optimal subsidies.** The figure depicts effects of the first-stage production technology, β_0 and the MVPF condition, m. The blue curve is the initial investment, I_0. The green (orange) curve is the later-stage investment following a high (low) signal, the red curve is the expected later-stage investment. The black curve is total expected investments, $\Psi(\alpha, \delta^*)$. The magenta curve is the subsidy rate scaled up by 1000. The panels use the base case from Table 1.

to a constant factor. This is not so when we consider the implications of the production technology and the MVPF.

To address the implications of the production technology we restrict our attention to the case in which the sum of the elasticities is kept constant. This restriction helps us to analyze whether subsidies affect firms differently depending on the firms' interdependency between the investments.

Figure 3(a) shows how the relative importance of the investments matters for the firm's investments. Intuitively, a higher elasticity on the initial investment makes the firm increase that investment whereas the expected later-stage investment decreases. Interestingly, according to (22) the government keeps the subsidy rate constant, and this is then not enough to prevent the total expected investments from decreasing.

Figure 3(b) examines the effects of a stricter MVPF requirement, m. An increase in m naturally results in a lower subsidy rate. The effect is substantial: raising m from 0.6 to 2 reduces the subsidy rate from approximately 0.68 to 0.33. Consequently, total expected investments decline from around 1800 to 275. As discussed previously, a more informative signal α will amplify these effects. Overall, the firm's investment policy is substantially influenced by the interplay of factors such as information quality, production technology, and subsidization constraints.

We can see the impact of the production technology directly from the ratio between expected later-stage investment and initial investments:

$$\frac{\mathbb{E}[I_1^*]}{I_0^*} = \frac{\beta_1}{\beta_0}. \tag{35}$$

This result follows directly from Corollary 2 and is not too surprising given the Cobb-Douglas styled value function. While this expected inter-temporal investment ratio is unaffected by all model parameters except the two elasticities, the actual investment outlays are directly affected by the information structure and the later-stage elasticity. We can see this by considering the ratio between later-stage investment outlays for the two possible signals:

$$\frac{I_1^*(H)}{I_1^*(L)} = \left[\frac{p(k|H)}{p(k|L)}\right]^{\frac{1}{1-\beta_1}}. \tag{36}$$

It is easily seen that this ratio increases in α, decreases in q, and increases in β_1. Remember that the signal is informative about the productivity parameter $x \in \{0, k\}$, in the sense that $\alpha > 1/2$ and that this means that $p(k|H) > p(k|L)$. Furthermore, since $\beta_1 \in (0, 1)$ it then follows that the ratio always will be at least 1. More importantly, as the signal's accuracy, α, increases, so does $p(k|H)$, while $p(k|L)$ decreases. Since $p(k|\cdot)$ is positive, it therefore follows from an application of the chain rule and the quotient rule that the ratio must be increasing. In combination with the analysis above, we can then see the full impact of signal accuracy: the overall expected investment level, $\Psi(\alpha, \cdot)$, increases and the investment strategy is such that a relatively large investment amount is allocated in the later-stage if and only if a positive signal, $s = H$, is received. The higher the accuracy, the more the decision problem thus resembles the exercise of a call option ("to invest"). The prior beliefs, q, have the opposite effect. As q increases, the direct impact of the signal–as analyzed via the updated (conditional) probabilities, $p(k|H)$ and $p(k|L)$–becomes less relevant for the firm's decision problem. Finally, as the elasticity of later-stage investments, β_1, increases, so does the investment ratio::

$$\frac{\partial}{\partial \beta_1}\left\{\frac{I_1^*(H)}{I_1^*(L)}\right\} = \frac{I_1^*(H)}{I_1^*(L)} \cdot (1 - \beta_1)^{-2} \cdot \ln \frac{p(k|H)}{p(k|L)}. \tag{37}$$

From this we can see that the investment ratio not only increases in β_1, but actually accelerates the impacts of q and α.

Below we address how these findings affect the society's value of subsidization and the firm's valuation of flexibility in investments.

3 Social net benefits and value of flexibility

Our model helps to understand the effects of governmental subsidization of a firm facing an investment problem with complementary investments. Importantly, our analysis takes into account that the government has a minimum requirement on the return its subsidization brings to society. To get further insights, we analyze the social net benefits in this framework as well as the implications for the firm's net return.

3.1 Social net benefits

The social net benefits consist of two sources: The firm's net present value of investments and the spillover effects from the firm's outcome and investments. We denote the former as the firm's Private Net Benefits, W, and with the government's subsidy policy δ^* it is given by

$$W = \mathbb{E}[V] - (1 - \delta^*)(I_0 + \mathbb{E}[I_1]). \tag{38}$$

The second source, the Government's Net Benefits, is denoted as G and is given by

$$G = \gamma_V \mathbb{E}[V] + \gamma_I(I_0 + \mathbb{E}[I_1]) - \delta^*(I_0 + \mathbb{E}[I_1]), \tag{39}$$

that is, the benefits of subsidies, $SubBen$, less the costs of subsidies. The sum, $W + G$, is the government's social net benefit which we denote as SNB, and we analyze how the MVPF, the production technology, and the information quality affect the social net benefit. We can derive how the subsidy policy and the resulting investments by the firm determine the private, government, and social net benefits.

Proposition 2 *The firm's Private Net Benefits, W, and the Government's Net Benefits,* *G, can be written as*

$$W = \Psi(\alpha, \delta^*)(1-\delta^*)\frac{1-\beta_0-\beta_1}{\beta_0+\beta_1} \qquad \text{and} \qquad G = \Psi(\alpha, \delta^*)\delta^*(m-1), \tag{40}$$

and thus

$$\text{SNB} = \frac{\Psi(\alpha, \delta^*)}{\beta_0+\beta_1}\Big(\delta^*(m-1)(\beta_0+\beta_1) + (1-\delta^*)(1-\beta_0-\beta_1)\Big). \tag{41}$$

We illustrate the effect of information quality in Figure 4(a). Intuitively, a higher level increases the private and government net benefits. This effect is due to the total expected investment level, $\Psi(\alpha, \delta^*)$. Figure 4(b) illustrates the effect of the initial investment's elasticity. As the subsidy rate is independent of the sum of the elasticities, we keep this sum constant. Again, the effect is due to the $\Psi(\alpha, \delta^*)$-channel. We note that the information quality has a substantial impact on the net benefits. This is particularly the case when the relative elasticity of the later-stage investment is high. We discuss this channel in more detail in Section 3.2.

In addition to the MVPF criterion the government is likely to prefer investments for which the social net benefits are positive. Rewriting the last factor in (41) we get

$$\text{SNB} = \frac{\Psi(\alpha, \delta^*)}{\beta_0+\beta_1}\Big([1-(\beta_0+\beta_1)] - [1-m(\beta_0+\beta_1)]\delta^*\Big), \tag{42}$$

whence it follows that SNB > 0 iff

$$1-(\beta_0+\beta_1) > [1-m(\beta_0+\beta_1)]\delta^*, \tag{43}$$

and inserting the expression for δ^* we obtain

$$[1-(\beta_0+\beta_1)](\gamma_V + m(\beta_0+\beta_1)) > [1-m(\beta_0+\beta_1)](\gamma_V + \gamma_I(\beta_0+\beta_1)), \tag{44}$$

thus

$$m > \frac{\gamma_V+\gamma_I}{1+\gamma_V+(\gamma_I-1)(\beta_0+\beta_1)} \triangleq m_0. \tag{45}$$

We observe that $m_0 > 0$ because $\gamma_I < 1$ and $\beta_0 + \beta_1 < 1$. A condition saying that the social net benefits must be positive is thus restricting the government to only consider subsidizing investments with not a too low MVPF. The threshold m_0 is increasing in the production technology parameters β_0 and β_1. This is not obvious from (41), but the intermediate result in (42) reveals that the effect stems from the subsidization channel. Indeed, δ^* decreases in the production technology parameters (because $m > \gamma_I$) and this effect dominates such that m^* is in fact increasing. We also note that

$$\frac{\partial m_0}{\partial \gamma_V} = \frac{(1-\gamma_I)(1-(\beta_0+\beta_1))}{(1+\gamma_V+(\gamma_I-1)(\beta_0+\beta_1))^2} > 0, \tag{46}$$

$$\frac{\partial m_0}{\partial \gamma_I} = \frac{1+\gamma_V(1-(\beta_0+\beta_1))}{(1+\gamma_V+(\gamma_I-1)(\beta_0+\beta_1))^2} > 0, \tag{47}$$

and hence

$$\frac{\partial m_0}{\partial \gamma_V} < \frac{\partial m_0}{\partial \gamma_I}, \tag{48}$$

because $\gamma_I < 1$ and $\beta_0 + \beta_1 < 1$ (and $\gamma_V > 0$).

It might seem surprising that the "SNB=0"-threshold increases in both spillover effects. This can be understood as follows. First, there is a direct positive effect of increasing the spillover effects on the MVPF, cf. (50) below. Second, there is an indirect effect through the substitution channel. Consider the MVPF for $\delta_0 = \delta_1 = \delta$:

$$MVPF = \frac{\gamma_V(1-\delta)(1-\delta)+\gamma_I[\beta_0(1-\delta)+\beta_1(1-\delta)]}{\delta\beta_0(1-\delta)+\delta\beta_1(1-\delta)} = \frac{\gamma_V(1-\delta)}{\delta(\beta_0+\beta_1)} + \frac{\gamma_I}{\delta}. \tag{49}$$

As expected, it follows that a higher substitution parameter decreases MVPF. From Proposition 1 we know that the optimal substitution rate increases in the spillover effects. This implies that there are two counterweighing forces at play. The net effect is parameter specific. However, we see from (43) that since δ^* increases in the spillover effect, so must the m that defines the SNP $= 0$ threshold. In addition, we also observe from (43) that higher spillover effects can affect SNP either negatively or positively. If m is small such that $m(\beta_0+\beta_1) < 1$, then a higher spillover effect decreases SNP. Reversely, íf m is high enough such that $m(\beta_0+\beta_1) > 1$, then a higher spillover effect increases SNP.

A cornerstone in our analysis is that information about the value of the project is only partially revealed before the later-stage investment. It is not obvious whether this affects the government's perception of supporting investments. In fact, we note that the MVPF for a general subsidization policy (δ_0, δ_1) is

$$MVPF = \frac{\gamma_V(1-\delta_0)(1-\delta_1) + \gamma_I[\beta_0(1-\delta_1) + \beta_1(1-\delta_0)]}{\delta_0\beta_0(1-\delta_1) + \delta_1\beta_1(1-\delta_0)}, \tag{50}$$

and hence the MVPF is independent of the information parameter α for any given subsidization policy. Since the government does not benefit from any slack in its MVPF-constraint, the government's optimal subsidization policy satisfies

$$MVPF = m, \tag{51}$$

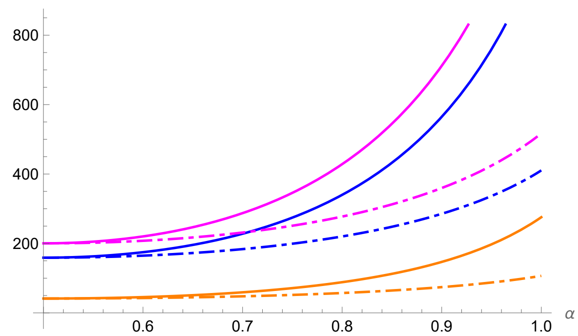
and thus it seems that the information quality, α, is unimportant to the government. However, assuming $m > m_0$ we see that SNB in (41) is increasing in the information parameter α through $\Psi(\alpha, \delta^*)$. Hence, it is important for a government to consider the information quality in a firm's investment problem.

Figure 4(c) illustrates the effect of the MVPF-threshold, m. We consider m higher than m_0. We observe that a higher threshold has opposite effects on the firm's net benefits and the government's net benefits. It follows from Proposition 1 that a higher threshold decreases the firm's subsidy rate, δ^*, and hence the total expected investments decrease, i.e., W decreases. From Proposition 2 we see that the government's net benefits consist of three factors. The first two factors, $\Psi(\alpha, \delta^*)$ and δ^*, are decreasing in m. The third factor, $m - 1$, increases in m. Thus, there is a non-monotonic effect in m. In Figure 4(c), the trade-off between the two forces changes when m is about 1.4.

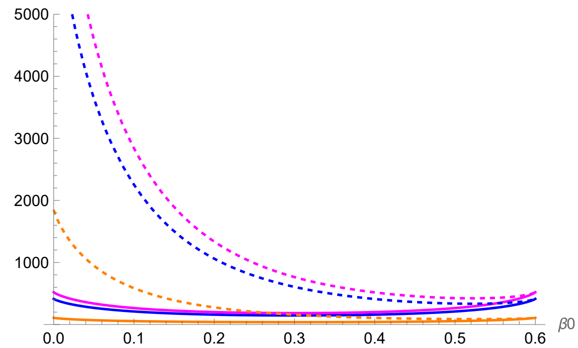
3.2 Option value

As our model pertains to subsidies that are intended to spur private investments it is essential that we develop a thorough understanding of the private firm's net return in context of these subsidies. Further, since we consider sequential investment decisions under the arrival of new information prior to the later-stage investment decision, it is also natural to investigate the relationship between the model's parameters and the firm's net returns as well as society's net expenditures.

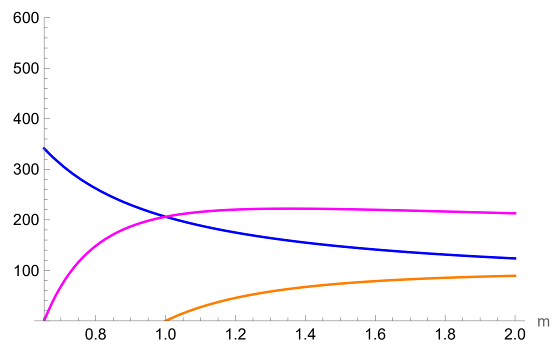
Note that the firm's expected net return without subsidies, $W|_{\delta=0}$, is positive. As



(a) Net benefits for varying α with fixed $\beta_1 = 0.6 - \beta_0$. Solid: Low initial investment elasticity ($\beta_0 = 0.2$). Dot-Dashed: High initial investment elasticity ($\beta_0 = 0.4$).



(b) Net benefits for varying β_0 fixing $\beta_1 = 0.6 - \beta_0$. Solid: No option value ($\alpha = 0.5$). Dashed: Normal option value ($\alpha = 1$).



(c) Net benefits for varying m. $\beta_0 = 0.2$.

Figure 4: **Net benefits with optimal subsidies.** The figure depicts effects of the information quality, α, the first-stage production technology, β_0, and the MVPF condition, m. The blue curve is the firm's private net benefits, W, the orange curve is the government's net benefits, G, and the magenta curve is the social net benefits, SNB, from Proposition 2. The panels use the base case from Table 1.

the government's goal in our model is to stimulate additional investments, the subsidy will effectively add to the firm's expected net return. That is, the expected subsidy, $\delta^*\Psi(\alpha,\delta^*)$, creates a value transfer of $W - W|_{\delta=0}$ to the firm. Proposition 2 tells us that this value transfer must be

$$W - W|_{\delta=0} = \frac{1-\beta_0-\beta_1}{\beta_0+\beta_1}\left[(1-\delta^*)\Psi(\alpha,\delta^*) - \Psi(\alpha,0)\right], \tag{52}$$

which can be re-written as:

$$W - W|_{\delta=0} = \frac{1-\beta_0-\beta_1}{\beta_0+\beta_1}\Psi(\alpha,0)\left[(1-\delta^*)^{\frac{-(\beta_0+\beta_1)}{1-\beta_0-\beta_1}} - 1\right] = W|_{\delta=0}\left[(1-\delta^*)^{\frac{-(\beta_0+\beta_1)}{1-\beta_0-\beta_1}} - 1\right], \tag{53}$$

and we can therefore interpret

$$(1-\delta^*)^{\frac{-(\beta_0+\beta_1)}{1-\beta_0-\beta_1}} - 1 \tag{54}$$

as the rate of value transfer to the firm.

As long as the signal $s \in \{L, H\}$ is informative (i.e., $\alpha > 1/2$), the flexibility to choose the later-stage investment level, I_1, will be valuable. This option value can be determined from Proposition 2 as $W|_{\alpha>1/2} - W|_{\alpha=1/2}$. Alternatively it can be measured in relative terms as the ratio of $W|_{\alpha>1/2}$ to $W|_{\alpha=1/2}$. However, we can take the analysis a step further and decompose the value that the firm receives from both information and subsidies in concert. As a base case, we let $\Psi_0 \triangleq \Psi(\frac{1}{2},0)$ be the expected investments when there is no information gain, $\alpha = 1/2$, nor subsidies, $\delta = 0$. Then

$$\Psi_0 = (\beta_0+\beta_1)\cdot(k\beta_0^{\beta_0}\beta_1^{\beta_1}q)^{\frac{1}{1-\beta_0-\beta_1}}. \tag{55}$$

With this definition, we can decompose the firm's value gains from information and subsidies. Formally we have:

Corollary 3 *The firm's gain of investment flexibility,* $W|_{\delta=\delta^*,\alpha>\frac{1}{2}} - W|_{\delta=0,\alpha=\frac{1}{2}}$, *consists of an information part*

$$W_{info} = W|_{\delta=0,\alpha>\frac{1}{2}} - W|_{\delta=0,\alpha=\frac{1}{2}}, \tag{56}$$

and a subsidiary part,

$$W_{subs} = W\big|_{\delta=\delta^*,\alpha>\frac{1}{2}} - W\big|_{\delta=0,\alpha>\frac{1}{2}}, \tag{57}$$

where

$$W\big|_{\delta=0,\alpha=\frac{1}{2}} = (1-\beta_0-\beta_1)\left[k\beta_0^{\beta_0}\beta_1^{\beta_1}q\right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{58}$$

$$W\big|_{\delta=0,\alpha>\frac{1}{2}} = \left(\frac{g^{1-\beta_1}}{q}\right)^{\frac{1}{1-\beta_0-\beta_1}} W\big|_{\delta=0,\alpha=\frac{1}{2}}, \tag{59}$$

$$W\big|_{\delta=\delta^*,\alpha>\frac{1}{2}} = (1-\delta^*)^{\frac{-(\beta_0+\beta_1)}{1-\beta_0-\beta_1}} W\big|_{\delta=0,\alpha>\frac{1}{2}}. \tag{60}$$

From Corollary 3 we learn that the firm's expected net return stems from a pure option value without subsidies, the direct value transfer due to the subsidy, and a base value:

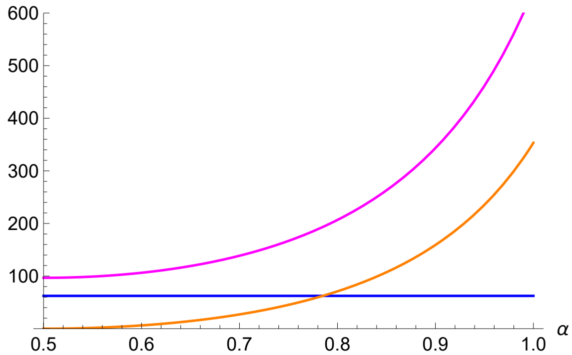
$$W\big|_{\delta=\delta^*,\alpha>\frac{1}{2}} = W_{info} + W_{subs} + W\big|_{\delta=0,\alpha=\frac{1}{2}}. \tag{61}$$

Furthermore, the net return can be factored out as

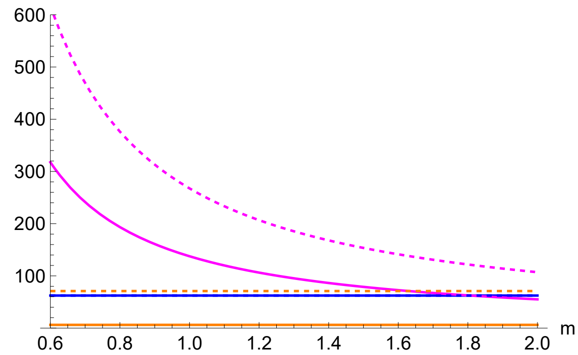
$$W\big|_{\delta=\delta^*,\alpha>\frac{1}{2}} = f_S(\gamma_V,\gamma_I,m,\beta_0+\beta_1)\cdot f_I(\beta_0,\beta_1,q,\alpha)\cdot f_P(\beta_0,\beta_1,k), \tag{62}$$

where the first factor, $f_S = (1-\delta^*)^{\frac{-(\beta_0+\beta_1)}{1-\beta_0-\beta_1}}$, is a subsidy effect. The second factor, $f_I = g^{\frac{1-\beta_1}{1-\beta_0-\beta_1}}$, stems from the value of information, whereas the third factor, $f_P = \frac{1-\beta_0-\beta_1}{\beta_0}\left[k\beta_0^{1-\beta_1}\beta_1^{\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}}$, is a productivity factor.

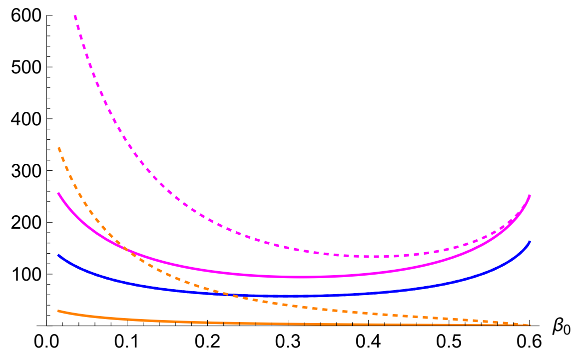
It follows directly from Proposition 1 that f_S is increasing in γ_V, γ_I, and $\beta_1+\beta_2$, yet decreasing in m. The reasons for this are straightforward: The market value spill-over (γ_V) and the investment spill-over (γ_I) both increase the subsidy effect. This happens directly via the increased subsidy. As the subsidy increases so does the overall investment level and hence the firm's net return. Similarly, when the minimum acceptable MVPF, (m), increases, the subsidy will decrease, which in turn will lower the overall investment level and hence lower to expected net return. Interestingly, the individual elasticities of capital, β_0 and β_1, do not affect the subsidy effect, only their sum, $\beta_0+\beta_1$, does. The reason for this is that the subsidy, δ^*, is independent of the relative elasticities as well, but determined by their sum. The subsidy effect is the only of the three factors that has this



(a) Firm NPV options for varying α. $\beta_0 = 0.2$.



(b) Firm NPV **options** for varying m, $\alpha = 0.8$ for dotted curves.



(c) Firm NPV options for varying β_0 with $\beta_0 + \beta_1 = 0.6$, $\alpha = 0.8$ for dotted curves.

Figure 5: **Firm value components.** Effects of information quality, production technology parameters and MVPF-threshold. The blue curve is $W|_{\delta=0,\alpha=1/2}$, the orange curve is W_{info}, the magenta curve is W_{subs}. The dashed curves use a higher information quality level $\alpha = 0.8$. The panels use the base case from Table 1.

property.

The information effect, f_I, decreases in both β_0 and β_1, while it increases in α and q. The fact the the information effect on the firm's net return increases in prior beliefs (q) as well as in the signal accuracy (α) is not surprising. Both of these parameters increases the investment levels and in turn increases the expected net return. The fact that the information effect decreases in both elasticities of capital, can be thought of as a form of dampening where high elasticities of capital investments lowers the relative importance of both prior and updated beliefs. The investment levels are less sensitive to all other models parameters as the elasticities increases. The analysis of the productivity effect, f_P, is analogous to that of the investment levels for varying models parameters.

Figure 5 illustrates the decomposition of real option values from Corollary 3, and their sensitivities to changes in information quality, investment elasticities, and the government's MVPF-threshold. Information quality contributes to the real option value via the direct information component, W_{info} (orange curve), as well as the subsidy component, W_{subs}, (magenta curve). The contribution from W_{info} reflects the value of investment flexibility. The contribution from W_{subs} stems from the fact that subsidies increase the overall investment level, which in turn increases the option value, see Figure 5(a). Furthermore, as indicated in Figure 5(b), W_{info} is unaffected by changes in the MVPF-threshold. In fact, the threshold exclusively affects the option value via W_{subs}, which decreases in m. The effect is more pronounced for higher information quality, due to the higher overall option value as noted earlier. Finally, as indicated in Figure 5(c), the impacts of changes in investment elasticities, β_0 and β_1, is less straightforward. In keeping with the previous analysis, we fix the sum of elasticities and vary the two βs simultaneously. From (58) it follows directly that the baseline value, $W|_{\delta=0,\alpha=\frac{1}{2}}$ (blue curve), is symmetrical and maximized near the two extremes, $\beta_0 = 0$ and $\beta_1 = 0$. This is due to the fact that we have decreasing returns to scale; as the value of the project becomes more dependent on a singular investment outlay, while the sum of elasticities are held fixed, the impact of the decreasing return to scale assumption becomes less important for the relevant investment outlay. On the other hand, when the two capital outlays are equally important, they are both subject to the decreasing returns to scale effect. In turn, the firm invests more in the extreme cases, and thus the baseline value increases near the extremes. The contribution from the information component is intuitively higher for a relatively higher β_1; this corresponds to a lower β_0 in Figure 5(c). As the information is received prior to the later-stage investment, this part of the real option value reaches its maximum when the later-stage investment is most important. This effect becomes more pronounced the higher the information quality is. The government can predict both of these effects, and the contribution from the subsidy component thus increases the option values particularly around the extreme β-values. It follows that for high elasticities of the later-stage investment, the subsidy component adds significantly to the option value.

4 Discussion

4.1 Subsidy risk

In many instances, subsidies are attached with some risk. For large capital intensive projects developed over several years, there will always be a risk that the sponsoring government (or other entity) will not commit to their initially announced subsidy policy. This risk naturally affects the firm's decision to invest, and in turn the government's announced subsidies. The simplest form of such a risk emerge when the government cannot commit due to an external factor that is entirely out of their control. In such cases the outcome is a subsidy announcement that compensates for this risk in a fashion similar to that of a bond issuer that compensates the market for credit-risk with a premium.

Formally, assume that there is a $d \in [0;1]$ probability that the government will provide the promised subsidy, and a $1-d$ probability that it will provide nothing. We assume that this risk materialize after the firm has made the investment, and that it is induced by an external circumstance that cannot be influenced by neither the government nor the firm. Furthermore, assume that this risk is independent of the productivity $x \in \{0, k\}$ as well as the signal s. In this setting, we can find the optimal investment levels, $\tilde{I}_0$ and $\tilde{I}_1(s)$ following the same steps as in Lemma 2. We find:

$$\tilde{I}_0 = \left[k \left(\frac{\beta_0}{1-\delta_0} \right)^{1-\beta_1} \left(\frac{\beta_1}{1-d\delta_1} \right)^{\beta_1} g^{1-\beta_1} \right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{63}$$

$$\tilde{I}_1(s) = \frac{p(k|s)^{\frac{1}{1-\beta_1}}}{g} \cdot \frac{\beta_1}{\beta_0} \cdot \frac{1-\delta_0}{1-d\delta_1} \cdot I_0^*. \tag{64}$$

The similarity with the result from Lemma 2 stems from the fact that the firm is risk neutral and that the subsidy risk does not materialize until after the firm has made the investment. This means that the firm cannot respond after having learned that no subsidy will be provided. The government shares the belief that there is a $1-d$ probability that it cannot deliver the subsidy as promised. This means that the government's objective function takes on the familiar form while the constraint is slightly altered to account for the eventuality that it will not provide any subsidies towards the end of the project. Therefore, the government's problem is:

$$\max_{\delta_0,\delta_1} \quad (1+\gamma_V)\mathbb{E}[V] - (1-\gamma_I)(\tilde{I}_0 + \mathbb{E}[\tilde{I}_1]) \tag{65}$$

s.t.

$$\frac{\gamma_V \mathbb{E}[V] + \gamma_I(\tilde{I}_0 + \mathbb{E}[\tilde{I}_1])}{\delta_0 \tilde{I}_0 + d\delta_1 \mathbb{E}[\tilde{I}_1]} \geq m. \tag{66}$$

We can solve this optimization following the same steps as in Lemma 3, and we find:

$$\tilde{\delta}_0 = d\tilde{\delta}_1 = \frac{\gamma_V + \gamma_I(\beta_0+\beta_1)}{\gamma_V + m(\beta_0+\beta_1)}. \tag{67}$$

Note that $\tilde{\delta}_0 = \delta^*$ while $\tilde{\delta}_1 = \delta^*/d$. This means that by introducing late stage subsidy risk in this fashion allows us to replicate the main results of the model exactly. Equivalently, we can interpret δ_1 as the expected late stage subsidy without any loss of generality.

This baseline result merits additional investigations both into the impact of subsidy risks with asymmetric information, moral hazard, as well as a deeper analysis of the strategic aspects of offering subsidies in uncertain environments. Cases of asymmetric information between the government and the firm as well as questions of moral hazard in the process when subsidies are awarded complicate the analysis considerably. It is beyond the scope of this paper to delve deep into the various setups where subsidy risks can be analyzed. However, they are subject to ongoing research and similar structures of the solutions can be identified. For instance, if we assume that the subsidy policy can be made contingent on the signal $s \in \{L, H\}$, such that $\delta_1(s) \in \{\delta_L, \delta_H\}$, and we keep the assumption that the government will commit to this subsidy policy, the firms investment levels can be identified using the same approach as in Lemma 2. We find,

$$\tilde{I}_0 = \left[k\left(\frac{\beta_0}{1-\delta_0}\right)^{1-\beta_1} \left(\frac{\beta_1}{1-\mathbb{E}[\delta_1]}\right)^{\beta_1} \tilde{g}^{1-\beta_1} \right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{68}$$

where

$$\tilde{g} \triangleq p_H p(k|H)^{\frac{1}{1-\beta_1}} \left(\frac{1-\mathbb{E}[\delta_1]}{1-\delta_H}\right)^{\frac{\beta_1}{1-\beta_1}} + p_L p(k|L)^{\frac{1}{1-\beta_1}} \left(\frac{1-\mathbb{E}[\delta_1]}{1-\delta_L}\right)^{\frac{\beta_1}{1-\beta_1}}. \tag{69}$$

Consequently, we find

$$\tilde{I}_1(s) = \frac{p(k|s)^{\frac{1}{1-\beta_1}}}{\tilde{g}} \cdot \frac{\beta_1}{\beta_0} \cdot \frac{1-\delta_0}{1-\mathbb{E}[\delta_1]} \left(\frac{1-\mathbb{E}[\delta_1]}{1-\delta_1(s)} \right)^{\frac{1}{1-\beta_1}} \tilde{I}_0. \tag{70}$$

The similarities with Lemma 2 are evident, but it is also clear that the full distribution of subsidy rates affects the firm's investment levels as well as their response to uncertainty (in this case modeled via $\tilde{g}$). This model is intriguing and allows us to investigate various economic problems related to investment spill over effects and the policy decisions. This analysis leverages the observation that expected spill over effects themselves depend on distribution of subsidy rates.[8]

4.2 Investments in signal accuracy, α

As in Flor and Grell (2013) we can expand on the current setting to endogenize the signal accuracy. A natural motivation for this extention is that governments play a dual role as subsidizing private investment as well as subsidizing society-wide research efforts (which in our case could influence the signal accuracy, α). The simplest way to endogenize this effect would be to add a cost of accuracy in the government's optimization problem. That is

$$\max_{\delta_0,\delta_1,\alpha} \quad (1+\gamma_V)\mathbb{E}[V] - (1-\gamma_I)(I_0 + \mathbb{E}[I_1]) - C(\alpha) \tag{71}$$

s.t.

$$\frac{\gamma_V \mathbb{E}[V] + \gamma_I(I_0 + \mathbb{E}[I_1])}{\delta_0 \tilde{I}_0 + \delta_1 \mathbb{E}[I_1]} \geq m, \tag{72}$$

$$\delta_0 I_0 + \delta_1 \mathbb{E}[I_1] + C(\alpha) \leq \Gamma, \tag{73}$$

where Γ reflects a joined budget for both direct subsidies and research support. The easiest way to make this extension is to introduce an increasing and convex cost function of α. However, it would be more interesting to consider an elaborate model in which α is an optimal outcome of an information creating technology. We leave such a complex extension

[8] This analysis is subject to ongoing research that builds on the extension presented in Subsection 4.3.

for future research. Alternatively, we could consider α as a function of the initial R&D, which would resemble the modeling of LP/GP games in venture capital firms as in Flor and Grell (2013).

Endogenous signals opens up another strand of inquiry. If the social benefits are known for various industries, the government's allocation of subsidies (and research support) will solve a first-order condition taking both the expected viability of the industry and the probability distribution of industry viability into account.

4.3 Alternative modeling of the information structure

Our analysis employs a binary information structure to clearly distinguish between a good and bad outcome. This makes the analysis tractable and ease interpretations, but since the production technology also depends on two complementary investments, the generality of the model's outcome is unclear. However, we are able to extend the model to a framework in which the return is log-normal and the signal has a continuous state space.

Specifically, we can let a be a normally distributed shock affecting the log-return

$$x = \exp\{A + a\}, \tag{74}$$

where $a \sim N(0, \sigma)$ and thus $A + \frac{1}{2}\sigma$ is the expected log-productivity.We assume the signal obtained after the initial investment is normally distributed with mean 0 and variance m. In Appendix 6.8 we show that the model's main results—Lemma 3 and Theorem 1—are still valid, and hence the subsequent analysis carry over to our alternative framework. In this framework, the informativeness of the signal, which is denoted α in the main model, is now related to the variance of the signal, m. Intuitively, as the variance m tends to zero, the signal is very informative about the shock adjustment a which corresponds to a higher α. Similarly, if the variance becomes very large, the signal is not very informative which corresponds to an α close to 1/2.[9]

[9] This interpretation assumes that the signal's variance changes as a free parameter. In the disclosure literature it is sometimes assumed that the total variance in the (log-)information system, $\sigma + m$, is kept constant (e.g., Christensen and Feltham, 2003). In this case it is desirable with a signal with a high variance as a realization then removes a large part of the total variance.

4.4 An application to green investments

According to the Climate Policy Initiative, annual green investment levels need to reach \$4.3 trillion (5% of global GDP) before 2030 in order to meet the goals of the Paris Agreement.[10] To accomplish this, investments need to grow more than 20% annually (year-over-year), and mechanisms for public and private capital supply are needed as well. For comparison, the accumulated green investment level between 2011 and 2020 was \$4.8 trillion. In this paper we analyze a model that can help predict how private investment decisions respond to public subsidy announcements. The path to a successful production using green technologies inherently involves investments in various stages. Such investments initially involve a research and development stage that, if successful, is followed by a growth stage. Our framework incorporates the interplay between investment flexibility, technology development, and information uncertainty. With this application in mind, subsidizing the green transition is most powerful when the return from sequential investments into a new green technology depends less on the research and development stage than on the subsequent growth stage. The ability to obtain new information during the investment sequence improves the effect of subsidies. Subsidizing information quality can dominate direct investment subsidies.

In a famous 2015 speech, then-Governor of the Bank of England, Dr. Mark Carney, described climate change as the "Tragedy of the Horizon."[11] The same speech advanced a framework for assessing climate change impacts on asset prices and the financial sector in three broad classes: i) Physical risks, pertaining to the actual damages stemming from climate change related events, such as extreme weather events, droughts, floods, or chronic worsening of infrastructure, buildings and similar; ii) Transition risks, experienced by individual sectors of the economy due to structural changes stemming from, for example, climate-related policies and, finally, iii) Liability risks, to the insurance industry from incremental claims of those harmed by climate change.

With this backdrop in mind, green investments are thus exposed to both climate risk

[10] Naran et al. (2022)

[11] Transcript: Carney (2015). Video available via https://www.bankofengland.co.uk/speech/2015/breaking-the-tragedy-of-the-horizon-climate-change-and-financial-stability (accessed 9/6/2024.)

directly as well as exposed to uncertainties around society's ability to quantify climate risks. Pindyck (2021) highlights how the uncertainty of economic impacts of climate change can be understood as a two-layered uncertainty that stems from incomplete information about the precise relationship between, for example, emissions and temperature on the one side, and the relationship between temperature increases and economic impacts on the other side.

A popular approach that incorporates the link between climate change and economic impacts is the Dynamic Integrated Climate-Economy (DICE) model (Nordhaus, 2018) which multiply a damage function (of measurable climate metrics) with an output function. Others look at the relationship between changes in climate variables and economic output without focusing on the damage function (Schlenker and Roberts, 2009; Dell et al., 2012; Burke et al., 2015, 2018; Colacito et al., 2019; Desmet et al., 2021). Weitzman (2011) underlines that economic welfare hinges on the degree to which climate change affects the probability distribution of future economic outcomes. Kiley (2021) shows that climate uncertainty itself has impacts on economic and financial stability.

The aforementioned types of risk hamper the willingness in the private sector to pursue green investments. In an effort to spur private sector green investments, governments subsidize projects at various states of development. Green investments differ from traditional investments insofar as companies are being incentivized to invest in the development of technologies that can help mitigate or perhaps eliminate climate related damages in cases where neither the marketability nor the relevance of said technologies have been proven. This presents an immense challenge from a capital budgeting point of view, because both initial and later-stage investment decisions are made under incomplete information. The traditional framework for growth options is therefore of limited use for green investments. In this context, we provide a structural model with two complementary investment outlays, where the assumptions about the arrival of information are lax enough to provide meaningful insights about the green investment problem. We have linked the accuracy of new information to initial and later-stage investment levels and leverage this link in a deeper analysis of optimal subsidies. Furthermore, we have analyzed the role of governments as providers of capital subsidies in a simple setting where the government has a constraint on the MVPF, which leads to the announcement of their relative capital subsidies.

5 Conclusion

This paper explores the dynamics between government subsidies, production technology, and learning, highlighting their collective impact on a firm's investment strategy. Our research underscores that the effectiveness of subsidies is not a straightforward function of the investment itself but is highly contingent on the interplay between the informativeness of signals received throughout the investment process and the specific characteristics of the production technology involved. A key finding is that more informative signals significantly enhance both governmental and corporate benefits, particularly when the production technology's returns rely more on later-stage investments. This suggests that subsidies are most effective when designed to account for the evolving information landscape, allowing firms to adapt their strategies as new knowledge emerges. The real options framework we develop extends traditional models by incorporating uncertainty at all stages of investment, providing a more robust tool for analyzing investment behavior under such conditions.

We also derive the optimal subsidization rate when a company is offered direct investment subsidies. This rate aligns a firm's intrinsic investment incentives with the dynamic nature of the learning process. This policy is particularly valuable in scenarios where the production technology exhibits higher elasticity in later-stage investments, as it allows firms to maximize their value by exploiting learning opportunities. Such schemes ensure that public funds are allocated to projects that maximize social net benefits and align with the private interests of firms, thereby fostering a more efficient allocation of resources. This study thus illustrates how government subsidies can be structured to support investment strategies that balance both public and private interests, particularly in environments characterized by uncertainty, imprecise learning, and the need for sequential investment decisions.

6 Appendix

6.1 Proof of Lemma 1.

Proof. Consider first the assertion

$$p(k|H) > p(k|L) \iff \alpha > 1/2. \tag{75}$$

Let α be given. From Bayes' Rule it follows that:

$$p(k|H) = p(H|k)\frac{p_k}{p_H} = \alpha\frac{q}{1-(\alpha+q)+2\alpha q}, \tag{76}$$

$$p(k|L) = p(L|k)\frac{p_k}{p_L} = (1-\alpha)\frac{q}{(\alpha+q)-2\alpha q}. \tag{77}$$

It follows that

$$p(k|H) > p(k|L) \iff \alpha\frac{q}{1-(\alpha+q)+2\alpha q} > (1-\alpha)\frac{q}{(\alpha+q)-2\alpha q} \tag{78}$$

$$\iff \alpha(\alpha+q-2\alpha q) > (1-\alpha)(1-(\alpha+q-2\alpha q)) \tag{79}$$

$$\iff 0 > (1-\alpha)-(\alpha+q-2\alpha q) \tag{80}$$

$$\iff 0 > (1-q)(1-2\alpha) \tag{81}$$

$$\iff \alpha > 1/2. \tag{82}$$

Similar calculations prove the assertion regarding conditional probabilities of $x = 0$. ■

6.2 Proof of Lemma 2.

Proof. We prove Lemma 2 by backward induction. The optimal later-stage investment level, $I_1^*(s)$ for $s \in \{L, H\}$, maximizes the expected value of V conditional on the signal, s, net of the later-stage investment level after subsidies; that is, $(1-\delta_1)I_1$. This means that:

$$I_1^*(s) = \arg\max\left\{p(k|s)kI_0^{\beta_0}I_1^{\beta_1} - (1-\delta_1)I_1\right\} = \left[\frac{k\beta_1 p(k|s)}{1-\delta_1}\right]^{\frac{1}{1-\beta_1}} I_0^{\frac{\beta_0}{1-\beta_1}} \tag{83}$$

Given these two potential later-stage investment levels, the initial investment, I_0, maximizes the expected net value of the later-stage investment, where expectations are taken over the potential signals, net of the initial subsidy. That is:

$$I_0^* = \arg\max \Big\{ p_H \cdot \Big[p(k|H) k I_0^{\beta_0} I_1^*(H)^{\beta_1} - (1-\delta_1) I_1^*(H) \Big] + \tag{84}$$

$$p_L \cdot \Big[p(k|L) k I_0^{\beta_0} I_1^*(L)^{\beta_1} - (1-\delta_1) I_1^*(L) \Big] - (1-\delta_0) I_0 \Big\} \tag{85}$$

which after some calculations yields

$$I_0^* = \left[k \left(\frac{\beta_0}{1-\delta_0} \right)^{1-\beta_1} \left(\frac{\beta_1}{1-\delta_1} \right)^{\beta_1} g^{1-\beta_1} \right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{86}$$

where

$$g = g(\alpha, q, \beta_1) \triangleq p_H p(k|H)^{\frac{1}{1-\beta_1}} + p_L p(k|L)^{\frac{1}{1-\beta_1}}. \tag{87}$$

Plug I_0^* into the expression for $I_1(s)$ to get the desired result. ■

6.3 Proof of Corollary 1

Proof. From Lemma 2 we get:

$$\frac{I_1^*(s)}{I_0^*} = p(k|s)^{\frac{1}{1-\beta_1}} \cdot \frac{\beta_1}{\beta_0} \cdot \frac{1-\delta_0}{1-\delta_1} \cdot g^{-1}. \tag{88}$$

By definition of g we get:

$$\frac{\mathbb{E}[I_1^*]}{I_0^*} = \frac{\beta_1}{\beta_0} \cdot \frac{1-\delta_0}{1-\delta_1}, \tag{89}$$

and (13) follows directly. Furthermore, from Lemma 2 we see that:

$$V(s) = x \cdot (I_0^*)^{\beta_0} \cdot (I_1^*(s))^{\beta_1} \tag{90}$$

$$= x \cdot p(k|s)^{\frac{\beta_1}{1-\beta_1}} \cdot \underbrace{\left[k^{\beta_0+\beta_1} \left(\frac{\beta_0}{1-\delta_0} \right)^{\beta_0} \left(\frac{\beta_1}{1-\delta_1} \right)^{\beta_1} g^{\beta_0} \right]^{\frac{1}{1-\beta_0-\beta_1}}}_{\triangleq h} \tag{91}$$

From here it follows that:

$$\mathbb{E}[V] = \mathbb{E}[x \cdot p(k|\cdot)^{\frac{\beta_1}{1-\beta_1}}] \cdot h \tag{92}$$

$$= \left(p_H \mathbb{E}[x \cdot p(k|H)^{\frac{\beta_1}{1-\beta_1}} | H] + p_L \mathbb{E}[x \cdot p(k|L)^{\frac{\beta_1}{1-\beta_1}} | L]\right) \cdot h \tag{93}$$

$$= \left(p_H \cdot p(k|H)^{\frac{1}{1-\beta_1}} + p_L \cdot p(k|L)^{\frac{1}{1-\beta_1}}\right) \cdot k \cdot h \tag{94}$$

$$= g \cdot k \cdot h \tag{95}$$

$$= \left[k \left(\frac{\beta_0}{1-\delta_0}\right)^{\beta_0} \left(\frac{\beta_1}{1-\delta_1}\right)^{\beta_1} g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{96}$$

and (14) follows directly. ■

6.4 Proof of Lemma 3

Proof. Using Lemma 2 we can rewrite the Government's objective function, (19), to:

$$\max_{\delta_0,\delta_1} \quad I_0^* \left[(1-\gamma_V)\frac{1-\delta_0}{\beta_0} - (1-\gamma_I) \cdot \left(1 + \frac{\beta_1}{\beta_0} \cdot \frac{1-\delta_0}{1-\delta_1}\right)\right], \tag{97}$$

which in turn can be re-written:

$$\max_{\delta_0,\delta_1} \quad \left[k \left(\frac{\beta_0}{1-\delta_0}\right)^{\beta_0} \left(\frac{\beta_1}{1-\delta_1}\right)^{\beta_1} g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}} \left[1 + \gamma_V - (1-\gamma_I)\left(\frac{\beta_0}{1-\delta_0} + \frac{\beta_1}{1-\delta_1}\right)\right]. \tag{98}$$

Similarly, we can re-write the constraint, (21), in the Government's optimization, by using Lemma 2 and the fact that $I_0^* > 0$. We get:

$$\gamma_V(1-\delta_0)(1-\delta_1) + \beta_0(1-\delta_1)(\gamma_I - \delta_0 m) + \beta_1(1-\delta_0)(\gamma_I - \delta_1 m) \geq 0 \tag{99}$$

From (98) and (99) the Lagrangian for the Government's decision problem is:

$$\mathscr{L}(\delta_0,\delta_1) = \left[k \left(\frac{\beta_0}{1-\delta_0}\right)^{\beta_0} \left(\frac{\beta_1}{1-\delta_1}\right)^{\beta_1} g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}} \left[1 + \gamma_V - (1-\gamma_I)\left(\frac{\beta_0}{1-\delta_0} + \frac{\beta_1}{1-\delta_1}\right)\right]$$
$$+ \lambda\left[\gamma_V(1-\delta_0)(1-\delta_1) + \beta_0(\gamma_I - \delta_0 m) + \beta_1(\gamma_I - \delta_1 m)\right].$$

First notice that $\frac{\partial \mathscr{L}}{\partial \lambda} = 0$ implies that:

$$\gamma_V(1-\delta_0)(1-\delta_1) = -\beta_0(1-\delta_1)(\gamma_I - \delta_0 m) - \beta_1(1-\delta_0)(\gamma_I - \delta_1 m). \tag{100}$$

Furthermore, $\frac{\partial \mathscr{L}}{\partial \delta_0} = 0$ implies that:

$$\left[k\left(\frac{\beta_0}{1-\delta_0}\right)^{\beta_0}\left(\frac{\beta_1}{1-\delta_1}\right)^{\beta_1} g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}} \cdot \left[\frac{\beta_0\left((\gamma_I-1)(\frac{\beta_0}{1-\delta_0}+\frac{\beta_1}{1-\delta_1})\gamma_V+1\right)}{1-\beta_0-\beta_1}+\frac{\beta_0(\gamma_I-1)}{1-\delta_0}\right] \tag{101}$$

$$= \lambda[\beta_1(\gamma_I - \delta_1 m) + \beta_0(1-\delta_1)m + (1-\delta_1)\gamma_V](1-\delta_0) \tag{102}$$

$$= \lambda[\beta_0(1-\delta_1)(m-\gamma_I)], \tag{103}$$

where the last equality follows from (100). Similarly, $\frac{\partial \mathscr{L}}{\partial \delta_1} = 0$ implies that:

$$\left[k\left(\frac{\beta_0}{1-\delta_0}\right)^{\beta_0}\left(\frac{\beta_1}{1-\delta_1}\right)^{\beta_1} g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}} \cdot \left[\frac{\beta_1\left((\gamma_I-1)(\frac{\beta_0}{1-\delta_0}+\frac{\beta_1}{1-\delta_1})\gamma_V+1\right)}{1-\beta_0-\beta_1}+\frac{\beta_1(\gamma_I-1)}{1-\delta_1}\right] \tag{104}$$

$$= \lambda[\beta_0(\gamma_I - \delta_0 m) + \beta_1(1-\delta_0)m + (1-\delta_0)\gamma_V](1-\delta_1) \tag{105}$$

$$= \lambda[\beta_1(1-\delta_0)(m-\gamma_I)], \tag{106}$$

where the last equality follows from (100). Combining these findings we see that:

$$\begin{aligned} &\lambda[\beta_1(1-\delta_0)(m-\gamma_I)] \cdot \left[\frac{\beta_0\left((\gamma_I-1)(\frac{\beta_0}{1-\delta_0}+\frac{\beta_1}{1-\delta_1})\gamma_V+1\right)}{1-\beta_0-\beta_1}+\frac{\beta_0(\gamma_I-1)}{1-\delta_0}\right] \\ = \quad &\lambda[\beta_0(1-\delta_1)(m-\gamma_I)] \cdot \left[\frac{\beta_1\left((\gamma_I-1)(\frac{\beta_0}{1-\delta_0}+\frac{\beta_1}{1-\delta_1})\gamma_V+1\right)}{1-\beta_0-\beta_1}+\frac{\beta_1(\gamma_I-1)}{1-\delta_1}\right], \end{aligned}$$

which reduces to $\delta_0^* = \delta_1^*$. Finally, since the Government has no benefit from slack in its constraint, we get:

$$\delta_0^* = \delta_1^* = \left[\frac{\gamma_V \mathbb{E}[V]}{I_0^* + \mathbb{E}[I_1^*]} + \gamma_I\right] \Big/ m \triangleq \delta^*. \tag{107}$$

Then from Corollary 1 the result, (22), follows directly. ■

6.5 Proof of Theorem 1

Proof. $\delta_0^* = \delta_1^* = \delta^*$ implies that

$$I_0^* = \left[\frac{k}{1-\delta^*}\beta_0^{1-\beta_1}\beta_1^{\beta_1}g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{108}$$

and from Lemma 3 it then follows that

$$I_0^* = \left[\frac{\gamma_V + m(\beta_0+\beta_1)}{(m-\gamma_I)(\beta_0+\beta_1)}\cdot k\beta_0^{1-\beta_1}\beta_1^{\beta_1}g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}}. \tag{109}$$

(28) and (29) follow directly from Lemma 2 applying $\delta_0^* = \delta_1^* = \delta^*$ as above. ■

6.6 Proof of Corollary 2

Proof. From (28) we get:

$$\mathbb{E}[I_1^*] = \underbrace{\mathbb{E}[p(k|s)^{\frac{1}{1-\beta_1}}]}_{\triangleq g}\cdot\left[\frac{\gamma_V + m(\beta_0+\beta_1)}{(m-\gamma_I)(\beta_0+\beta_1)}k\beta_0^{\beta_0}\beta_1^{1-\beta_0}g^{\beta_0}\right]^{\frac{1}{1-\beta_0-\beta_1}} \tag{110}$$

$$= \left[\frac{\gamma_V + m(\beta_0+\beta_1)}{(m-\gamma_I)(\beta_0+\beta_1)}k\beta_0^{\beta_0}\beta_1^{1-\beta_0}g^{1-\beta_1}\right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{111}$$

which was to be shown. From (29) we get:

$$\mathbb{E}[V] = \mathbb{E}[x\cdot p(k|\cdot)^{\frac{\beta_1}{1-\beta_1}}]\cdot\left[\left(\frac{\gamma_V + m(\beta_0+\beta_1)}{(m-\gamma_I)(\beta_0+\beta_1)}k\right)^{\beta_0+\beta_1}\beta_0^{\beta_0}\beta_1^{\beta_1}g^{\beta_0}\right]^{\frac{1}{1-\beta_0-\beta_1}}, \tag{112}$$

where

$$\mathbb{E}[x\cdot p(k|\cdot)^{\frac{\beta_1}{1-\beta_1}}] = p_H\mathbb{E}[x\cdot p(k|\cdot)^{\frac{\beta_1}{1-\beta_1}} \mid H] + p_L\mathbb{E}[x\cdot p(k|\cdot)^{\frac{\beta_1}{1-\beta_1}} \mid L] \tag{113}$$

$$= k\cdot g, \tag{114}$$

and (32) follows directly. ■

6.7 Proof of Lemma 4

We first show that g increases in α. Afterwards, we show that g increases in q and decreases in β_1.

g increases in α:

Proof. To ease the analysis we rewrite $g(\alpha, q, \beta_1)$ as

$$g = q^{\frac{1}{1-\beta_1}} \left(P_H \left[\frac{\alpha}{P_H}\right]^{\frac{1}{1-\beta_1}} + P_L \left[\frac{1-\alpha}{P_L}\right]^{\frac{1}{1-\beta_1}} \right) \tag{115}$$

Using (33) and (34) we calculate

$$\frac{\partial g}{\partial \alpha} = \frac{q^{\frac{1}{1-\beta_1}}}{1-\beta_1} \left[\left[\frac{\alpha}{P_H}\right]^{\frac{\beta_1}{1-\beta_1}} - \left[\frac{1-\alpha}{P_L}\right]^{\frac{\beta_1}{1-\beta_1}} - \beta_1(2q-1) \left(\left[\frac{\alpha}{P_H}\right]^{\frac{1}{1-\beta_1}} - \left[\frac{1-\alpha}{P_L}\right]^{\frac{1}{1-\beta_1}} \right) \right] \tag{116}$$

It is easily verified that $\alpha = 1/2$ implies that $P_H = P_L$, and thus $\frac{\partial g}{\partial \alpha} = 0$. Assume therefore that $\alpha \in (1/2; 1]$. Furthermore, if $q \leq 1/2$ then $2q - 1 \leq 0$ and $P_H \leq 1/2 \leq P_L$ such that $\frac{\alpha}{P_H} > \frac{1-\alpha}{P_L}$, and thus $\frac{\partial g}{\partial \alpha} > 0$. We therefore assume that $q > 1/2$.

Define k_1 and k_2 such that:

$$\frac{P_H}{\alpha} = 2q - 1 + \tfrac{1-q}{\alpha} \triangleq k_1(2q-1) \tag{117}$$

$$\frac{P_L}{1-\alpha} = 2q - 1 + \tfrac{1-q}{1-\alpha} \triangleq k_2(2q-1) \tag{118}$$

Note that $1 < k_1 < k_2$. With these observations, we rewrite (116) to:

$$\frac{\partial g}{\partial \alpha} = \frac{q^{\frac{1}{1-\beta_1}}}{1-\beta_1} \left[\left[\frac{\alpha}{P_H}\right]^{\frac{\beta_1}{1-\beta_1}} \left(1 - \beta_1 \frac{2q-1}{2q-1+\frac{1-q}{\alpha}}\right) - \left[\frac{1-\alpha}{P_L}\right]^{\frac{\beta_1}{1-\beta_1}} \left(1 - \beta_1 \frac{2q-1}{2q-1+\frac{1-q}{1-\alpha}}\right) \right] \tag{119}$$

$$= \frac{q^{\frac{1}{1-\beta_1}}}{1-\beta_1} \left[[(2q-1)k_1]^{\frac{-\beta_1}{1-\beta_1}} \left(1 - \frac{\beta_1}{k_1}\right) - [(2q-1)k_2]^{\frac{-\beta_1}{1-\beta_1}} \left(1 - \frac{\beta_1}{k_2}\right) \right] \tag{120}$$

$$= \frac{q^{\frac{1}{1-\beta_1}}}{1-\beta_1} (2q-1)^{\frac{-\beta_1}{1-\beta_1}} \left[k_1^{\frac{-1}{1-\beta_1}} (k_1 - \beta_1) - k_2^{\frac{-1}{1-\beta_1}} (k_2 - \beta_1) \right], \tag{121}$$

which is positive if and only if $\left[\frac{k_2}{k_1}\right]^{\frac{1}{1-\beta_1}} > \frac{k_2-\beta_1}{k_1-\beta_1}$, which is equivalent to:

$$(1-\beta_1)\ln(k_1 - \beta_1) - \ln(k_1) > (1-\beta_2)\ln(k_2 - \beta_1) - \ln(k_2), \tag{122}$$

which is true since the function $f(x) = (1-\beta_1)\ln(x - \beta_1) - \ln(x)$ is decreasing in x for $x > 1$ and $\beta_1 \in (0; 1)$. ■

g increases in q:

Proof. Simple partial differentiation yields:

$$\frac{\partial g}{\partial q} = \frac{1}{(1-\beta_1)q} \cdot \Big[\Big(p(k|H)^{\frac{1}{1-\beta_1}} - p(k|L)^{\frac{1}{1-\beta_1}} \Big) \cdot (2\alpha - 1) \cdot (1-\beta_1) q \tag{123}$$

$$+ (1-\alpha) p(k|H)^{\frac{1}{1-\beta_1}} + \alpha p(k|L)^{\frac{1}{1-\beta_1}} \Big], \tag{124}$$

which is positive because $\alpha \geq 1/2$ and $0 < p(k|L) \leq p(k|H)$ ■

g decreases in β_1:

Proof. Simple partial differentiation yields:

$$\frac{\partial g}{\partial \beta_1} = \frac{1}{(1-\beta_1)^2} \cdot \left[P_H p(k|H)^{\frac{1}{1-\beta_1}} \ln(p(k|H)) + P_L p(k|L)^{\frac{1}{1-\beta_1}} \ln(p(k|L)) \right], \tag{125}$$

which is negative since $p(k|s) \leq 1$ for $s \in \{L, H\}$ with equality at the most for one of the two probabilities. ■

6.8 Signal following a normal distribution

We now change the assumption that the signal has a binary distribution. Specifically, we assume as before that the investments yield a terminal value of

$$V = x I_0^{\beta_0} I_1^{\beta_1}, \tag{126}$$

but now the uncertain total productivity of capital has a lognormal distribution:

$$x = \exp\{A + a\}, \tag{127}$$

where $A + \frac{1}{2}\sigma^2$ is the expected log-productivity, and a is ex ante risk component in the productivity. a follows a normal distribution with mean 0 and variance $\sigma > 0$. The firm's signal, s, is informative about a:

$$s = a + \varepsilon, \tag{128}$$

where ε has a normal distribution with mean 0 and variance $m > 0$. We observe that the signal's distribution is normal with mean 0 and variance $\sigma + m$. It follows that the conditional distribution of the risk component, $a|s$, is also normal with mean $\frac{\sigma}{\sigma+m}s$ and variance $\frac{m}{\sigma+m}\sigma$.

We want to show that Lemma 3 and Theorem 1 are valid with the alternative information structure (with a reformulation of constants). To do so we first derive the investment levels and we note that

$$\mathbb{E}[x|s] = \mathbb{E}[\exp\{A + a\}|s] = \exp\{A + \frac{\sigma}{\sigma + m}(s + \tfrac{1}{2}m)\}, \tag{129}$$

and hence the last investment level given the signal solves

$$I_1^*(s) = \arg\max_{I_1} \left\{ I_0^{\beta_0} I_1^{\beta_1} \exp\{A + \frac{\sigma}{\sigma + m}(s + \tfrac{1}{2}m)\} - (1 - \delta_1) I_1 \right\}. \tag{130}$$

The first-order condition implies that

$$I_1^*(s) = \left(\frac{\beta_1}{1 - \delta_1} \exp\{A + \frac{\sigma}{\sigma + m}(s + \tfrac{1}{2}m)\} \right)^{\frac{1}{1-\beta_1}} I_0^{\frac{\beta_0}{1-\beta_1}}. \tag{131}$$

Furthermore,

$$\mathbb{E}[x I_1^{\beta_1}|s] = \mathbb{E}[x|s] I_1(s)^{\beta_1},$$

and after some calculations we get

$$\mathbb{E}[x I_1^{\beta_1}|s] = \exp\left\{ \frac{1}{1 - \beta_1} \left(A + \frac{\sigma}{\sigma + m}(s + \tfrac{1}{2}m) \right) \right\} \left(\frac{\beta_1}{1 - \delta_1} \right)^{\frac{\beta_1}{1-\beta_1}} I_0^{\frac{\beta_0\beta_1}{1-\beta_1}}. \tag{132}$$

This implies that we after several manipulations can obtain

$$\begin{aligned}
\mathbb{E}[x I_1^{\beta_1}] &= \mathbb{E}[\mathbb{E}[x I_1^{\beta_1}|s]] \\
&= \exp\{A + \frac{\sigma}{\sigma + m} \frac{1}{2} \left(m + \frac{\sigma}{1 - \beta_1} \right) \}^{\frac{1}{1-\beta_1}} \left(\frac{\beta_1}{1 - \delta_1} \right)^{\frac{\beta_1}{1-\beta_1}} I_0^{\frac{\beta_0\beta_1}{1-\beta_1}} && (133) \\
&= æ^{\frac{1}{1-\beta_1}} \left(\frac{\beta_1}{1 - \delta_1} \right)^{\frac{\beta_1}{1-\beta_1}} I_0^{\frac{\beta_0\beta_1}{1-\beta_1}}, && (134)
\end{aligned}$$

where we introduce the constant æ to shorten the notation. We also need to consider

$$\mathbb{E}[I_1] = \mathbb{E}[\mathbb{E}[I_1|s]],$$

which after manipulations become

$$\mathbb{E}[I_1] = æ^{\frac{1}{1-\beta_1}} \left(\frac{\beta_1}{1-\delta_1}\right)^{\frac{\beta_1}{1-\beta_1}} I_0^{\frac{\beta_0}{1-\beta_1}}. \tag{135}$$

The initial investment level solves

$$I_0^* = \arg\max_{I_1} \left\{\mathbb{E}\left[xI_0^{\beta_0} I_1^{\beta_1} - (1-\delta_1)I_1 - (1-\delta_0)I_0\right]\right\}. \tag{136}$$

Using the first-order condition, the previous expressions, and by reducing the outcome we obtain

$$I_0^* = \left(æ\left(\frac{\beta_0}{1-\delta_0}\right)^{1-\beta_1} \left(\frac{\beta_1}{1-\delta_1}\right)^{\beta_1}\right)^{\frac{1}{1-\beta_0-\beta_1}}. \tag{137}$$

Inserting (137) in (131) and manipulating the outcome gives us

$$I_1(s) = \exp\left\{\frac{1}{1-\beta_1}\frac{\sigma}{\sigma+m}\left(s - \tfrac{1}{2}\frac{\sigma}{1-\beta_1}\right)\right\}\frac{\beta_1}{\beta_0}\frac{1-\delta_0}{1-\delta_1}I_0. \tag{138}$$

Thus I_0^* and $I_1(s)$ have the same form as in Lemma 2. Moreover, we can use this to obtain (after some manipulations) that

$$\mathbb{E}[I_1^*] = \frac{\beta_1}{\beta_0}\frac{1-\delta_0}{1-\delta_1}I_0. \tag{139}$$

Similarly, we can derive value of the firm after the signal. Following a number of reductions we get

$$V(s) = \mathbb{E}[V|s] = \exp\left\{\frac{1}{1-\beta_1}\frac{\sigma}{\sigma+m}\left(s - \tfrac{1}{2}\frac{\sigma}{1-\beta_1}\right)\right\}\frac{1-\delta_0}{\beta_0}I_0, \tag{140}$$

and it follows that

$$\mathbb{E}[V] = \frac{1-\delta_0}{\beta_0}I_0. \tag{141}$$

This shows that we in this setup also get a result similar to Corollary 1. It follows that Lemma 3 and Theorem 1 are also valid under the alternative information structure.

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